Measurement of Bandwidth of Microwave Resonator by Phase Shift of Signal Modulation*

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Summary—Bandwidth is measured by transmission of a signal with sine-wave modulation through a microwave resonator under test. The modulation frequency is adjusted so that the envelope is delayed 45° with respect to the input, indicating that the two sideband frequencies are separated by the half-power bandwidth. The resonance ratio (Q) is then equal to the ratio of carrier frequency over twice the modulation frequency. This depends on observations of these frequencies and the modulation phase shift, but not on the amplitude. It is insensitive to detuning or incidental frequency variation of the resonator or the signal. In a resonant cavity tested, an observed bandwidth of 30 kc at 700 mc indicated that $Q=23{,}300$.

THIS PAPER describes a technique for measuring the bandwidth of a high-Q resonator by observing the transmission phase shift of the envelope of a modulated signal. Furthermore, this technique is suited for microwave distributed-circuit measurements as distinguished from techniques which are suited only for lumped-circuit measurements at lower frequencies. In general, prior methods have depended on amplitude observations, whereas the subject method does not.

Microwave resonators typically have a high Q, much greater than 1000. They are utilized in narrow-band filters, wavemeter cavities, and echo boxes. The common method [1], [2] of measuring the bandwidth would require an observation of the resonance curve, noting the frequency difference between the half-power (3-db) points on the sides. Much ingenuity has been applied to this method, such as sweeping the curve, much expanded, on a scope and then superimposing amplitude and frequency markers for ease and precision of observation.

A special method of microwave measurement, commonly applied to the echo box, relies on observing the envelope of the damped free oscillation following excitation by a short pulse. This also relies on observation of amplitude, particularly its rate of decay with time.

In a lumped resonant circuit, the "Q meter" relies on directly observing the ratio of the voltages across one reactance and the net resistance [3], [4]. It relies on amplitude measurement also, and, in addition, it is not generally applicable to distributed-circuit resonators.

As an introduction to the principle of the subject method, Fig. 1 shows the well-known behavior of transmission through a resonator at frequencies near resonance. The upper curve (a) shows the amplitude peak at f_0 and the two side points separated by the half-power

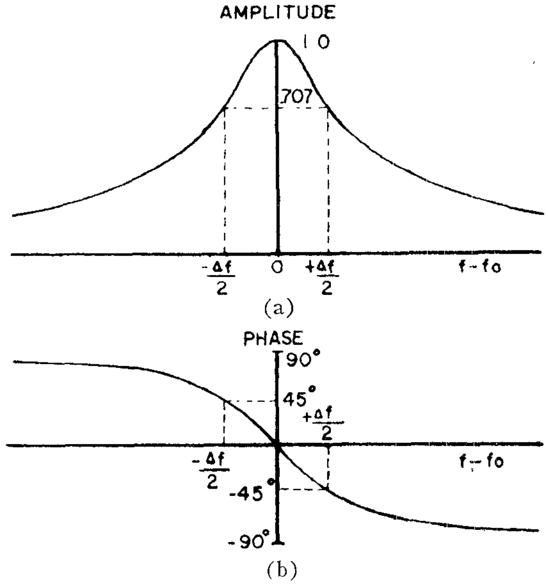

Fig. 1—Transmission characteristics of single-tuned resonator.

(3-db) bandwidth Δf. The lower curve (b) shows the phase slope and the corresponding two side points at $\pm 45°$ relative to the center.

If a carrier with sine-wave amplitude modulation is transmitted through such a resonator, with the carrier at the resonator center frequency and the sidebands at the half-power frequencies, the modulation envelope is delayed by 45° at the output. This phase lag with respect to the input results from the 45° phase shift at each of the sideband frequencies relative to that at the carrier frequency.

The block diagram of Fig. 2 shows the means of utilizing this principle for determining bandwidth. The signal generator provides an RF carrier with sidebands to give a sine-wave envelope of amplitude modulation. This is transmitted through the RF resonator under test. The input and output signals of the resonator are rectified in a detector, and the resulting modulation waves are amplified. The two resulting sine waves are applied to a phase meter to indicate their phase difference. An auxiliary amplitude indicator is provided for assuring that the amplitude is within the wide limits of normal operation of the phase meter.

In operation, the signal is tuned to the resonator center frequency and the modulation frequency is adjusted to give an indication of 45° on the phase meter. The details of this procedure will be given after a further description of the equipment.

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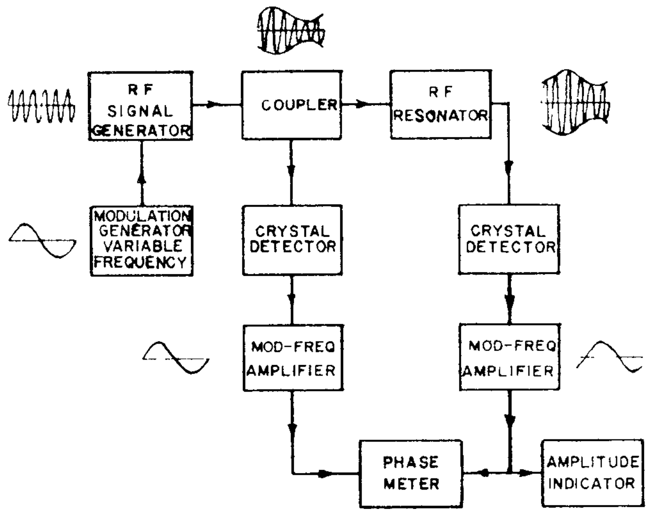

Fig. 2—Block diagram of measurement equipment.

In the signal generator, the carrier frequency must be tunable to the resonator frequency, and should be stable within about 1/10 of the bandwidth being measured. The signal generator must have provision for sine-wave modulation. This is readily accomplished internally in the types of circuits commonly used in signal generators below 1000 mc. At higher frequencies, where internal linear amplitude modulation may not be available, a crystal or magnetic modulator may be inserted in the output of the signal generator. The modulation factor should be kept moderately low (say less than 0.3) so as to hold down any envelope distortion in modulation, which may be a cause of error.

The modulation generator must provide a sine wave of a frequency which will be made equal to one-half the bandwidth. A continuously adjustable frequency is provided because the setting of the modulation frequency is part of the measurement procedure.

The coupler provides a reference signal by sampling the input to the cavity. Its coupling loss should approximately equal the insertion loss of the cavity. If the coupler is not directional, the line from coupler to resonator should be padded so that the reflections from the resonator will not cause errors.

The detectors and amplifiers are similar, and must have the same phase shift at the modulation frequency. The detectors may have any law of rectification that is not too unusual. The measurement procedure corrects for errors caused by distortion in detection, provided that the distortion is not so large as to change the number or the sequence of the zero crossings. The modulation-frequency amplifiers are needed to provide a sufficiently high amplitude for the operation of the phase meter.

The phase meter may be of any (within limits) of the several commercial types which indicate relative phase angle independent of amplitude and frequency [5]. Such a meter may rely on the zero crossings of the two applied signals, as will be assumed here.

The amplitude indicator is an oscilloscope or a voltmeter which indicates the output from the resonator. It is required merely to indicate whether the amplitude is within the operating limits of the phase meter.

In operation, the output of the RF signal generator is applied to the resonator under test. A sample of the applied signal is coupled to the first crystal detector, and serves as a reference. The applied signal is transmitted through the cavity and thereby has its modulation envelope changed. The modulation factor is reduced and the phase angle of the modulation is retarded; only the latter change is used for the measurement. The transmitted signal is detected and amplified like the reference signal. Both signals are applied to the phase meter, which then indicates directly their phase difference. The amplitude indicator may be used for setting the initial tuning in addition to monitoring the signal level to insure that it is within the range specified for the phase meter.

The measurement procedure is as follows:

a) The modulation frequency is set at some value less than the bandwidth to be measured, so that the detected modulation can be used for setting the carrier frequency.
b) The carrier frequency is set by adjusting the RF signal generator for maximum transmission through the resonator, with the aid of the amplitude indicator. This operation may be made completely independent of amplitude by tuning the RF signal generator for maximum phase shift instead of maximum amplitude. These alternative procedures yield the same result.
c) The frequency of the modulation generator is adjusted for 45° phase shift as indicated by the phase meter. The effects of any detector distortion can be corrected by making two measurements, as discussed below.
d) The bandwidth is equal to the frequency difference between sidebands, or twice the modulation frequency. The resonance ratio (Q) is therefore the carrier frequency divided by twice the modulation frequency.

The wave forms involved in the phase measurement are shown in Fig. 3. The like sine waves (a) illustrate the modulation signals applied to the phase meter in the absence of distortion. The time intervals marked 45° indicate the intervals used for the phase indication. The unlike wave forms (b) represent the outputs of detectors having different laws of rectification. The distortion does not change the time of positive and negative peaks of the signal, but it does change the time of the zero crossing. For a sine-wave modulation envelope, this distortion is symmetrical about the peaks. That is, if one zero crossing is advanced, the other is retarded by an equal amount. This is illustrated by the two time intervals which deviate from 45° by $\pm e$. By using the average of two phase measurements, using respectively

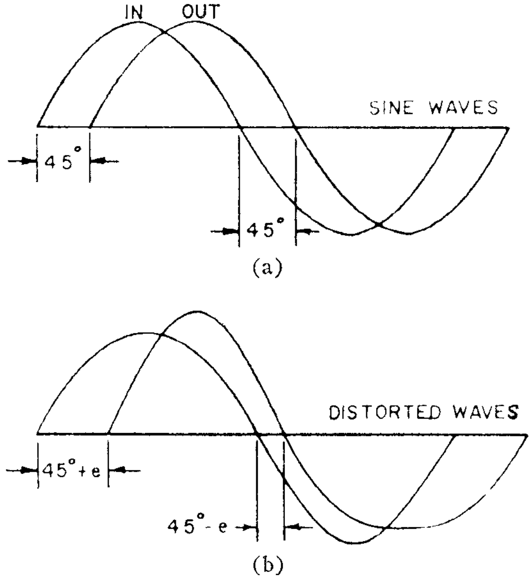

Fig. 3—Detected waveforms of modulation envelope.

the positive-going and the negative-going zero crossings, the effects of a moderate amount of distortion may be cancelled. For the particular phase meter used, this is accomplished by interchanging the two inputs. In general, the two measurements may be made by reversing the polarity of both inputs.

The bandwidth is indicated directly as twice the modulation frequency. This is inherently more accurate than separately measuring the half-power frequencies and then subtracting these frequencies to evaluate their small difference. Amplitude variations of the signal generator have no effect, since the measurement utilizes phase angle only. The envelope phase shift through the resonator has its greatest value with the carrier at the resonant frequency. Therefore, deviations from this frequency have only a second-order effect on phase shift, and the measurement is relatively insensitive to frequency drift of the signal generator and the resonator under test. A small amount of frequency modulation at the amplitude modulation frequency does not impair the accuracy, because its sidebands do not affect the detected amplitude.

For example, measurements by this procedure have been used to determine a bandwidth of 30 kc at a frequency of 700 mc, which gives a Q of 23,300. The accuracy is estimated to be within 3 per cent, corresponding to an error of 0.7° in measuring the phase. If individual half-power frequencies were measured to obtain this result, they would have to be determined within one part in 2 million. As compared with other techniques that are available for determining the bandwidths of high-Q resonators, the modulation phase method offers advantages of simplicity in operation and accuracy of determination.

Bibliography

[1] M. Wind and H. Rapaport, "Handbook of Microwave Measurements," Microwave Res. Inst., Polytechnic Inst. of Brooklyn, Brooklyn, N. Y.; 1954. (Section V, "Measurement of Q," mentions phase shift of 45° at half-power frequencies.)

[2] C. G. Montgomery, "Technique of Microwave Measurements," Rad. Lab. Ser., McGraw-Hill Book Co., Inc., N. Y., chs. 5 and 6; 1947.

[3] F. E. Terman, "Radio Engineers' Handbook," McGraw-Hill Book Co., Inc., New York, N. Y., p. 916; 1943.

[4] "Instructions and Manual of RF Measurements for the Q-Meter Type 100A," Boonton Radio Corp., Boonton, N. J.; March, 1935. (Talk by C. J. Franks and W. D. Loughlin at Rochester Fall Meeting, Rochester, N. Y.; November, 1934.)

[5] Y. P. Yu, "Coincident slicer measures phase directly," *Electronics*, vol. 31, pp. 99–101; September 12, 1958. (Description of phase meter.)

[6] D. C. King, "Measurements at Centimeter Wavelength," D. Van Nostrand Co., Inc., New York, N. Y.; 1952. (Resonant circuits, pp. 128–147.)

[7] E. L. Ginzton, "Microwave Measurements," McGraw-Hill Book Co., Inc., New York, N. Y., ch. 9; 1957. (Measurement of Q, various methods.)

[8] F. H. James, "A method for the measurement of very high Q-factors of electromagnetic resonators," Proc. IRE, vol. 106, pp. 489–492; September, 1959. (Recent proposal of the subject method.)